UDC 621.385.624

BBC • ITA

Engineering Research Report

The work covered by this report was undertaken by the BBC Research Department for the BBC and the ITA

Reducing non-linear effects in klystrons for television broadcasting

No. 1972/20

Research Department, Engineering Division
THE BRITISH BROADCASTING CORPORATION

RESEARCH DEPARTMENT

REDUCING NON-LINEAR EFFECTS IN KLYSTRONS FOR TELEVISION BROADCASTING

Research Department Report No. 1972/20
UDC 621.385.624

It uses SI units in accordance with B.S. document PD 5686.

P. Rainger

Head of Research Department

A.G. Lyner, B.Sc.

(RA-97)

Research Department Report No. 1972/20

REDUCING NON-LINEAR EFFECTS IN KLYSTRONS FOR TELEVISION BROADCASTING

(RA-97)

June 1972

Research Department Report No. 1972/20
UDC 621.385.624

REDUCING NON-LINEAR EFFECTS IN KLYSTRONS FOR TELEVISION BROADCASTING

Summary

After a brief discussion of klystron operation, a transfer characteristic is derived for a klystron. The effects of the non-linearity on a combined television sound and vision signal are discussed. A simplified klystron transfer characteristic is derived and a design of circuit is suggested for narrow band pre-correction of the input signal to the klystron for elimination of the effects of its non-linear behaviour on a television signal.

1. Introduction

Klystron amplifiers currently used for UHF television broadcasting from main transmitting stations and from translator stations are, at present, severely underrun when used to transmit sound and vision signals simultaneously, in order to reduce the effects of non-linear distortion to acceptable levels. This is extremely inefficient and a method of using present devices at higher output powers would be highly desirable.

The mechanism of amplification in a klystron depends on the dynamic behaviour of a beam of electrons, and is fundamentally non-linear, so that a change of klystron design would not bring about any improvement.

The long group delay through a klystron prohibits the use of negative feedback to reduce non-linear distortion. Two further methods remain which can be used to improve the linearity of the device; pre-distortion of the input signal by a non-linear law which is the inverse of the klystron non-linear law, and post-correction where a low level correcting signal is added to the transmitter output to cancel the distortions generated by the klystron.

In this report a transfer characteristic for a simple two-cavity klystron is derived using a ballistic theory already justified by other authors.[1,2,3,4] Ideally the effects of non-linearity could be overcome by pre-correcting the input signal with a suitable circuit having the inverse of this transfer characteristic. Complete pre-correction would only be possible in a system with a very large bandwidth and, is therefore not applicable to a klystron because of the band limiting effect of the tuned cavities which form part of the amplifier and couple with the electron beam. Methods are discussed, which can reduce the levels of those few in-band products which prove most objectionable, to the level of the next most noticeable effects, thus permitting an increase in the useable power of existing transmitters.

2. Transfer characteristic of a two-cavity klystron

Briefly a two-cavity klystron operates as follows.[2,3,4,9] An electron gun generates a uniform beam of electrons (typically $2A$ at $20\,kV$) which passes through the gap in the input cavity (Fig. 1). A coupling loop introduces signal energy into the cavity which produces a magnified electric field across the gap in the direction of the beam. The velocity of the electron beam is modulated by the r.f. field across the gap and, as the electrons drift along a field free region the faster ones begin to overtake the slower ones and bunching occurs.* When the bunched beam passes the output cavity the r.f. energy associated with the current variations is coupled into the cavity and a loop couples this energy into an output feeder.

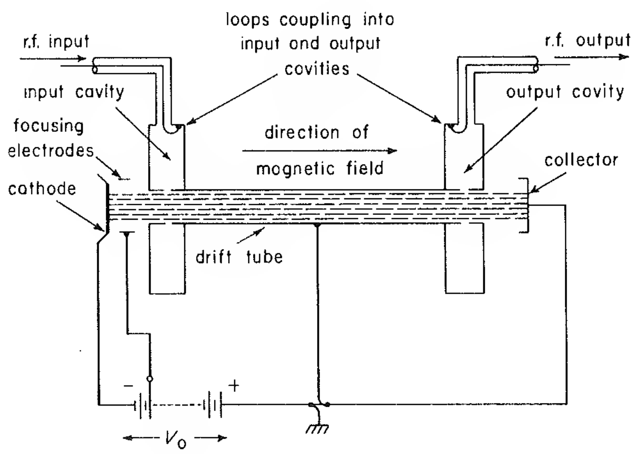

Fig. 1 - A two-cavity klystron

* The longitudinal magnetic field used to focus the beam does not affect the forward velocity of the beam.

(RA-97)

3. Ballistic analysis of the electron beam

In Appendix A a transfer function for a velocity modulated electron beam is derived from simple ballistic theory. Equations (A(11)) and (A(12)) show the output current $I(\tau)$, at time τ_1 in terms of the input signal $V_1 x(t)$ at time t. For a sinusoidal input where $x(t)$ = sin ωt Equations (A(11)) and (A(12)) may be written:

$$I(\tau) = Io/(1 - K \cos \omega t) \qquad (1)$$

and

$$\tau = t - \frac{K}{w} \sin \omega t \qquad (2)$$

where K is the so-called bunching parameter which is proportional to the amplitude of the input signal, and Io is the mean beam current.

Fig. 2 shows how, for a sinusoidal input, the output current $I(t)$ of the klystron beam may be derived by applying the solution of (2), graphically to (1) to eliminate the variable τ.

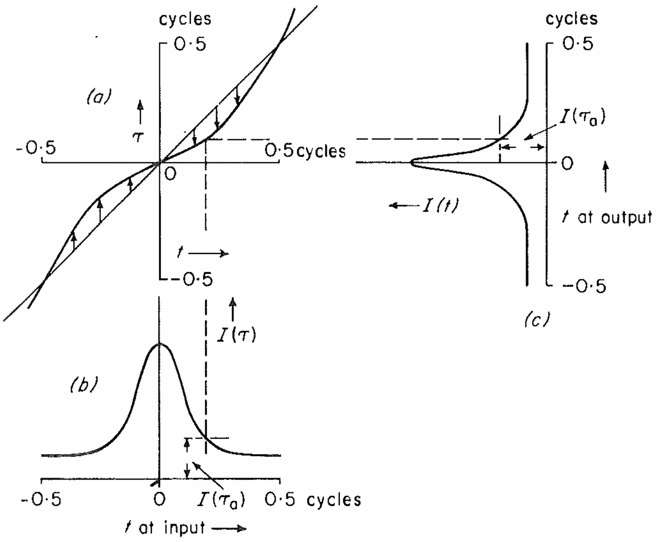

Fig. 2 - The current waveform at the klystron output cavity for sinusoidal input (no overbunching of electrons)

(a) Shows the relationship between τ and t for a sinusoidal drive

$$\tau = t - \frac{K}{\omega}\sin\omega t$$

(b) Shows how $I(\tau)$ varies with time at the input (Eq. 1)

(c) Shows the actual output current $I(t)$ measured at the output cavity found by combining graph (a) with graph (b)

For a sinusoidal input where $K>1$ (2) will become multivalued and this corresponds to some overtaking of electrons, causing 'over bunching' of the beam. Equations (1) and (2) must then be modified to account for this as $I(\tau)$ will comprise contributions from more than one portion of the input signal. Equations (1) and (2) may be combined to give a fuller description of the behaviour of the electron beam for any value of K.

$$I(\tau) = \sum_n \left| \frac{I_0}{1 - K \cos \omega t_n} \right| \qquad (3)$$

where t_n are roots of

$$\tau = t - \frac{K}{\omega} \sin \omega t$$

The modulus sign in (3) is needed as electrons arriving at the output in reverse order from that in which they left the input will still register as a positive current through the gap. The solution to Equation (2) will have either single values, or odd numbers of values, the number of solutions increasing as K increases. In practice K never exceeds a value where there are more than three solutions. Figure 3 shows the output current for a sinusoidal input when I is a little less than 1·5.

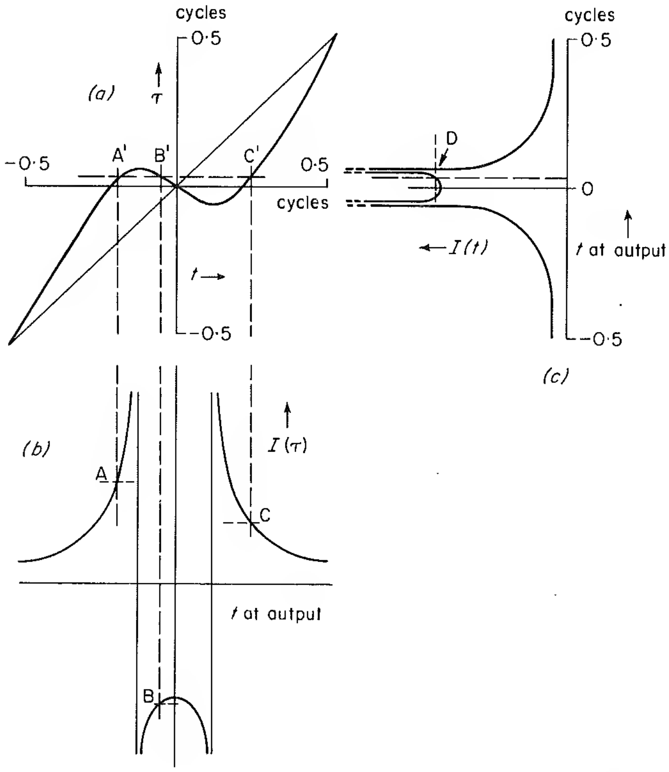

Fig. 3 - Overbunching at the output for a large amplitude sinusoidal input

(a) Shows the multiple values of τ as a function of t

(b) Shows how $I(\tau)$ varies with t

(c) Shows the net result of the contributions from (b) to the total output current

Currents at times A, B and C in 3(b) contribute to the current at time D in 3(c). A', B' and C' in 3(a) show the multiple solution of t for one value of τ (at D)

By an extension of this argument, for a general input function, as discussed in Appendix A, the transfer equation of the electron beam becomes, by modifying (A12):

$$I(\tau) = \sum_n \left| \frac{I_0}{1 - \frac{2\Delta}{u_0}\dot{x}(t)_n} \right| \qquad (4)$$

where t_n are roots of

$$\tau = t - \frac{2\Delta}{u_0} x(t)$$

In principle Equation (4) describes the ballistic behaviour of the beam however much 'over bunching' occurs although the output waveform may not be readily deduced because of the intrinsic time relationship.

Because the transfer equation of the klystron is implicit in time it does not give a readily understandable description of the distortion caused by a klystron. The output waveform may be more conveniently expressed as a series of Fourier components once the form of input waveform has been specified. This has in fact been done by several authors for a single tone* input and for the sum of two and three tones.[3,6,9]

For a single tone input, the output $I(\tau)$ may be written

$$I(\tau) = I_o a_o + I_o \sum_{r=1}^{\infty} \left[a_r \cos(r\omega\tau) + b_r \sin(r\omega\tau) \right] \qquad (5)$$

$$\text{Where } a_r = \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{I(\tau)}{I_o} \cos(r\omega\tau)\, d(\omega\tau) \qquad (6)$$

and

$$b_r = \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{I(\tau)}{I_o} \sin(r\omega\tau)\, d(\omega\tau) \qquad (7)$$

By changing the integral variable in (6) and referring to (3)

$$a_r = \frac{1}{\pi} \int_{-\pi}^{\pi} \sum_n \left| \frac{1}{1 - K\cos\omega t_n} \right| \cos\left\{ r(\omega t_n - K \sin \omega t_n) \right\} (1 - K\cos\omega t_n) d(\omega t_n) \qquad (8)$$

If bunching is limited so that electrons contributing to one cycle of signal on the beam, are conserved within the cycle throughout the bunching process, then all solutions for

$$\tau = t_n - \frac{K}{\omega} \sin \omega t_n \qquad (9)$$

lie within $-\pi \leqslant \omega t_n \leqslant \pi$ and so (8) may be written as

$$a_r = \frac{1}{\pi} \int_{-\pi}^{\pi} \cos\left\{ r(\omega t_n - K \sin \omega t_n) \right\} d(\omega t_n) \qquad (10)$$

which gives

$$a_r = 2J_r(rK) \qquad (11)$$

Similarly $b_r = 0$ and $a_o = 1$

So (3) may be expressed as

$$\frac{I(\tau)}{I_o} = 1 + 2 \sum_{r=1}^{\infty} J_r(rK) \cos(r\omega\tau) \qquad (12)$$

Equation (12) has been plotted by several authors[3,9] for a number of values of K (see Fig. 4).

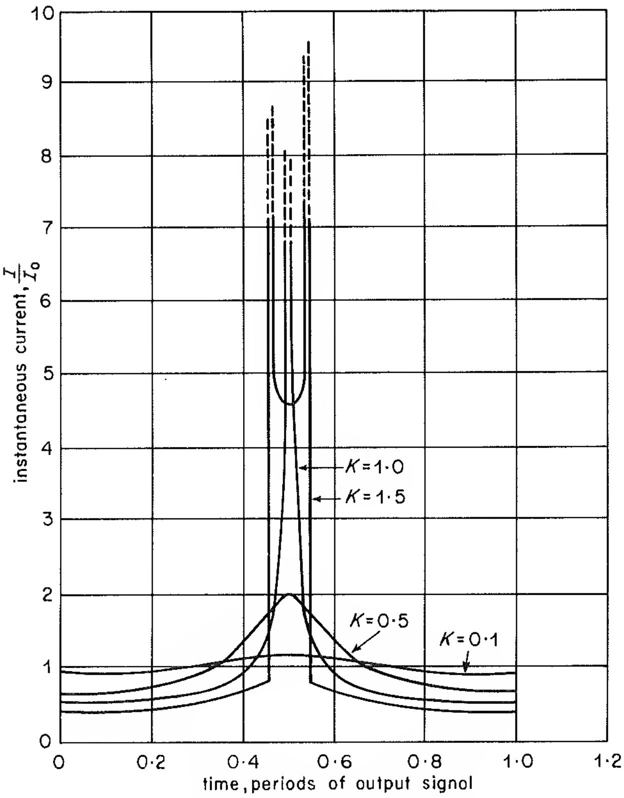

Fig. 4 - Waveform of current entering output cavity of two-cavity klystron

By a similar approach, it has been shown that,[6] for a three tone input, the beam at the output contains frequency components with amplitudes proportional to

$$a_{mnp} = J_m(\alpha).J_n(\beta).J_p(\gamma) \qquad (13)$$

where

$$\left.\begin{aligned} \alpha &= K_1(mf_1 + nf_2 + pf_3)/f_1 \\ \beta &= K_2(mf_1 + nf_2 + pf_3)/f_2 \\ \gamma &= K_3(mf_1 + nf_2 + pf_3)/f_3 \end{aligned}\right\} \qquad (14)$$

m, n, p, are integers, positive or negative and f_1, f_2, f_3 are frequencies of the input tones. K_1, K_2, K_3 are the 'bunching parameters' of the three signals, defined as for Equation 12 with a value which would apply if each tone were applied in the absence of the other two.

The mutual repulsion of electrons in the beam, which has been neglected in this analysis, has been found to leave

* 'tone' is used as a convenient term for a single-frequency r.f. signal.

the form of the output very little different from that described by Equation 13 except that the values of the bunching factors K_1, K_2, K_3, need slight modification.[3] The de-bunching of the beam due to space charge shortens the effective lengths of the drift spaces.

N.B. In the case of a single tone where $K_2 = K_3 = n = p = 0$, Equation 13 reduces to

$$a_{moo} = J_m\,(mK)$$

which compares with the coefficients of Equation (12).

The saturated output power occurs where the output at the fundamental frequency is at a maximum. This is when $J_1\,(K)$ is a maximum, i.e. when $K = 1{\cdot}84$ and $J_1\,(K) = 0{\cdot}582$. It is convenient to express the levels of inputs and outputs relative to this value. The output powers at various frequencies generated by a three tone input can be written as:

$$\left(\frac{P_{mnp}}{P_o}\right)^{\frac{1}{2}} = \frac{F_{mnp}}{{\cdot}582}\, J_m(\alpha)\, J_n(\beta)\, J_p(\gamma) \qquad (15)$$

with α, β and γ re-defined as

$$\left.\begin{aligned} \alpha &= 1{\cdot}84\,K_1\,(mf_1 + nf_2 + pf_3)/f_1 \\ \beta &= 1{\cdot}84\,K_2\,(mf_1 + nf_2 + pf_3)/f_2 \\ \gamma &= 1{\cdot}84\,K_3\,(mf_1 + nf_2 + pf_3)/f_3 \end{aligned}\right\} \qquad (16)$$

where K_1, K_2, K_3 are the normalised bunching parameters, P_o and P_{mnp} are saturated output power and power at $mf_1 + nf_2 + pf_3$ respectively and F_{mnp} is a factor depending upon the amplitude/frequency characteristics of the klystron cavities.

4. The effect of cavities. Multicavity klystrons

Whereas Equation (4) describes the non-linear behaviour of the electron beam, the complete transfer characteristic of the klystron is affected by the frequency response of the tuned cavities. Non-linear products generated by the electron beam will be coupled, to varying degrees, by the output cavity to the output of the device, depending on their frequency. Harmonics and products outside the pass-bands* of the cavities will not be coupled to the beam. In order to increase the bandwidth and gain of a klystron amplifier it is normal practice to cascade two or three amplifying sections along the same electron beam by means of intermediate cavities resistively loaded and stagger-tuned. The r.f. power level in the final drift space is higher than in the preceding spaces and it is there that the main effects of non-linearity occur. It is sufficient in fact to consider the final drift space as being the only non-linear part of the amplifier. However, the frequency response of the linear section prior to the penultimate cavity must be taken into consideration when calculating intermodulation products in the final drift space as the signal spectrum there will differ from that at the klystron input.

* The harmonics of the beam may well not coincide with higher order modes in the cavities.

5. The effects of non linearity upon the TV signal[3,4,5,7,8]

For television transmissions, klystron amplifiers are used in either of two ways. At main stations separate klystron amplifiers are used to amplify the vision signal and sound signal to the required output power levels before they are combined in a passive network into a common aerial system. The primary effect in the vision amplifier in this case is the crushing of the chrominance signal by the stronger luminance signal. In a colour picture this causes the colour saturation of an object to vary as its brightness varies and transmitters are run at about 2 dB lower than saturated power level to reduce the effect to an acceptable level.

In relay stations and, under standby conditions at some high power stations when a sound or vision klystron has failed, a single klystron is used to amplify both sound and vision signals together. Under this conditions, the non-linearity introduces distortions into the signal at lower operating powers. The most objectionable effects are caused by intermodulation products, and occur at

1) The sound carrier frequency,

2) $f_v + f_s - f_c$ at 1·57 MHz above vision carrier and

3) $2f_c - f_s$ at 2·85 MHz above vision carrier;

where f_v, f_s and f_c are the frequencies of vision carrier, sound carrier and colour subcarrier respectively.

1) corresponds to a crushing of the sound carrier by the vision carrier and manifests itself as amplitude modulation of the sound carrier by the line and field synchronising pulses. In some domestic receivers, the f.m. sound section cannot reject high levels of amplitude modulation and a buzz on the output can be heard. (This buzz, due to sound crushing in the klystron, is not to be confused with similar effects due to receiver non-linearity.)

2) and 3) are interference patterns on the picture which constantly change in phase and amplitude as the sound and picture change. Of these, 2) is at the higher level and is most noticeable.

Assessment of television transmitter amplifiers used to broadcast a combined sound and vision signal is frequently made using a three tone test.[8] Three frequencies, corresponding to vision carrier, sound carrier and colour subcarrier are injected into the klystron at input levels of −8 dB, −7 dB and −17 dB relative to peak sync power. If the intermodulation product at $f_v + f_s - f_c$ is less than −52 dB relative to peak sync power then, experience has shown, the impairment will be subjectively imperceptible under most picture conditions. To reduce the level of the $(f_v + f_s - f_c)$ product to the acceptable level of −52 dB the common sound and vision amplifier must be derated by some 7 dB more than for vision amplification alone. This reduces the efficiency of the amplifier to about 6% and it would be a great advantage if a method were developed to increase the available power from transmitters without raising the level of the non-linear products.

6. Possibilities of reducing non-linear effects at high power

The length of delay introduced by a klystron amplifier is too great for the use of negative feedback to linearise its behaviour. Two possibilities remain; post-correction and pre-correction of the signal.

Post-correction of the klystron by adding a correcting signal to the output would be one solution to the problem of non-linear distortion, but the realisation in practice of such a system would be difficult. The correcting signal would have to be added to the klystron output in the correct phase over the bandwidth used and in a way which involved no appreciable loss of transmitted power. Whilst this method cannot be rejected totally, it is thought that pre-correction would be easier and less costly to achieve and it is with pre-correction that the latter part of this report deals.

7. Possibilities of pre-correction

Ideally, the range of output power of a klystron could be extended by pre-distorting the input waveform by means of a circuit possessing the inverse of the non-linear transfer function of the klystron (see Appendix B). The input waveform necessary to produce a sinusoidal output from the klystron is derived in Appendix C and Figure 5 shows that the required input waveform would contain a number of high-frequency components to achieve high output amplitudes. The input and intermediate cavities would severely alter the relative amplitudes and phases of their components. Consequently, the possibility of overall pre-correction of the signal with a wideband pre-corrector must be dismissed.

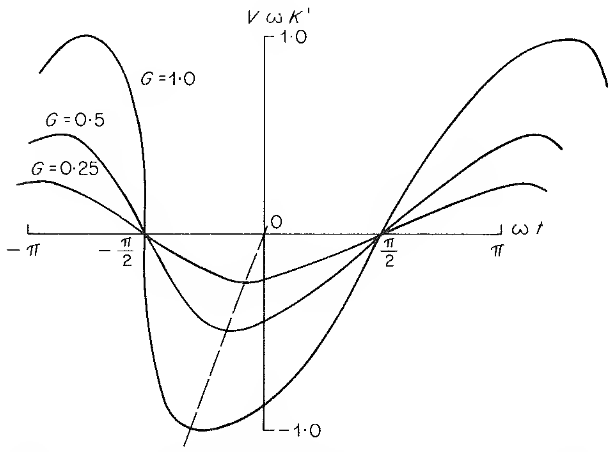

Fig. 5 - The input waveform necessary to produce a sinusoidal output of various amplitudes

As complete overall pre-correction would seem to be impracticable the possibility of partial pre-correction is now discussed. Basically, the only troublesome effects of non-linearity are in-band intermodulation products, as any out of band products can be controlled by the use of output filters. All products of a measureable level which fall within band must be odd-order and the three most offending products are those described in Section 4.

For pre-correction it is necessary that the input signal should have, added to it, components of equal magnitude and opposite phase to those generated by the klystron. Pre-correction for total cancellation of all in-band products would not be possible but, if the three most objectionable intermodulation products could be significantly reduced, the output power of the klystron could be raised until a new set of products produced noticeable impairment.

These three products could either be generated separately and added to the signal or the whole signal could be passed through a network which would produce the necessary pre-distortion. The latter, would be preferable provided that it is possible to control independently the phases of the products so generated.

Reference to the transfer function (Equation 4) of the klystron shows that the non-linearity is not generally in power series form, viz.

$$V_{out}(t) = a + b\,V_{in}(t) + c\,V_{in}(t^2) + d\,V_{in}(t^3) + \ldots \text{ etc.} \qquad (17)$$

The offending in-band products, however, which are third order* can be generated by a static non-linearity which involves no higher than a 5th degree polynomial with suitable coefficients. The function $J_1(1{\cdot}84K_1)/{\cdot}582$ gives the input-output law of the fundamental frequency for a single tone input (see Equation (15)) and, for $0<K_1<1$ this can be approximated to within ±1% by

$$\left(\frac{P}{P_o}\right)^{1/2} = 1{\cdot}58K_1 - {\cdot}6613K_1^{\,3} + {\cdot}0797K_1^{\,5} \qquad (18)$$

It can be easily shown that, to produce a power transfer characteristic of polynomial form:–

$$\left(\frac{P}{P_o}\right)^{1/2}_{out} = aK_1 + a_2K_1^{\,3} + a_3K_1^{\,5} \qquad (19)$$

the required instantaneous voltage transfer characteristic is

$$V_{out} = G\left\{1a_1V_{in} + \frac{4}{3}.a_2V^3 + \frac{8}{5}.a_3V^5 + \ldots\right. \qquad (20)$$

where 1, 4/3, 8/5 etc. are the so-called 'degeneracy' co-efficients and G is the overall gain. A transfer characteristic producing the fundamental input/output characteristic of Equation (18) would consequently be

$$V_{out} = G\left\{1{\cdot}58K_1 - 0{\cdot}885K_1^{\,3} + 0{\cdot}1278K_1^{\,5}\right\} \qquad (21)$$

As well as producing a single tone power in/power out law approximating to that of a klystron, a non-linearity of the form of Equation (21) would produce in band distortions closely similar to those produced by a klystron on a television signal or three-tone test signal. A pre-corrector with the inverse of this law would produce distortions which would cancel those generated by the klystron. Equation (22) gives a pre-corrector law which would effectively linearise the system.

$$V_{out} = G'(1{\cdot}58K_1 + 0{\cdot}885K_1^{\,3} - 0{\cdot}276K_1^{\,5}) \qquad (22)$$

(G' being the gain or loss of the pre-corrector)

* See Section 4.

It is possible that the phases of the intermodulation products generated by the pre-corrector may need some adjustment to achieve a high degree of cancellation. The exact nature of the phase adjustment required is not known. It is possible that a simple variable delay may be sufficient for good cancellation but a variation of phase across the video band may be found necessary.

In assessing the performance of a pre-corrector at U.H.F. it is more convenient to observe the intermodulation products it generates from a three tone signal than to measure its input output law. This is possible as the levels of the intermodulation products relate to the coefficients of the polynomial transfer characteristic.

Having arrived at a form of circuit that can be expected to have the correct sign of curvature to its transfer characteristic, study of its action upon a three tone signal will then give a convenient assessment of its performance prior to its being used in conjunction with a klystron.

8. Forms of pre-correction circuit

A circuit in the form shown in Fig. 6 would produce a distortion of the general form required (i.e. a stretching of the signal at large amplitudes).

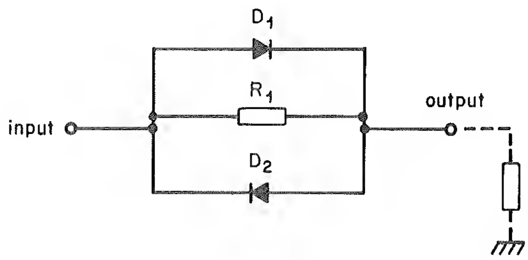

Fig. 6 - Simple form of pre-corrector to 'stretch' the input signal at high levels

This simple circuit has a number of limitations however. The knee voltage of available diodes is limited to the region of a few hundred millivolts and the forward resistance is, at best, a few tens of ohms. If the correcting device is to be placed between the driver amplifier and the klystron, powers of up to 1 watt will need to pass through it. The degree of 'bending' achievable with the circuit is not sufficient, whatever impedance the circuit is designed to have.

Using transistors as in Fig. 7, to perform the same function as the diodes in Fig. 6 a power in/power out law can be achieved with a knee at about 100 mW. An experimental circuit of this type has been built and has been found to produce an in-band intermodulation product corresponding to $(f_v + f_s - f_c)$ at a level of −40 dB (relative to a notional peak sync power of 100 mW) in a three-tone test. Ideally, this product would be of opposite phase to that which the klystron would generate and the resulting output of the amplifier should contain the intermodulation

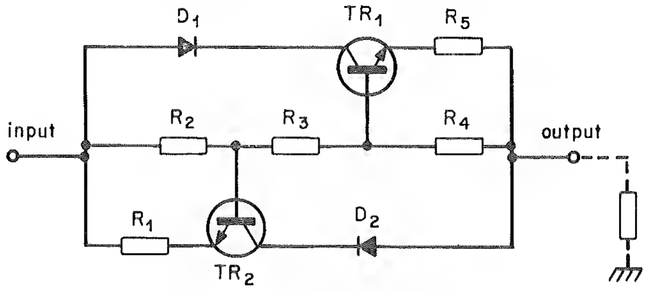

Fig. 7 - Circuit with similar characteristics as in Fig. 6 but with 'higher' knee voltage achieved by using transistors

product at a greatly reduced level due to partial, if not total, cancellation. By introducing reactive elements into the circuit of Fig. 7 it should be possible to achieve a variation in both phase and amplitude of non-linear products. Some flexibility could be achieved in the law of the device by preceding and following it with variable attenuators with complementary characteristics so that the corrector worked at a variable level whilst the input level to the kly-worked at a variable level whilst the input level to the klystron remained unchanged. A more complex system which would provide more flexible pre-correction is shown in Fig. 8. This would enable the phases of the products, generated by a non-linear circuit, to be varied with respect to the main signal.

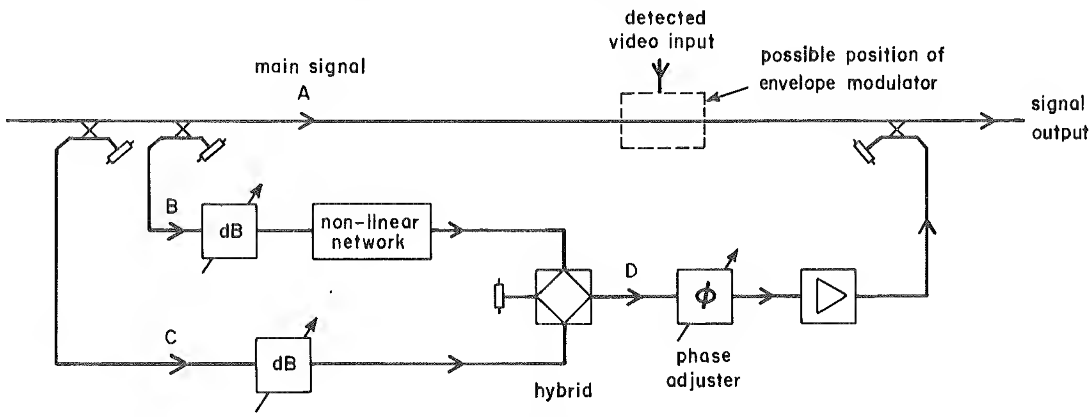

Fig. 8 - Possible pre-correction circuit with phase adjustment. Portions B and C are taken from the main signal A. B passes through a non-linearity and is subtracted from C using a hybrid ring and suitable attenuators to leave only the non-linear products. The phases of these products may then be adjusted and, after amplification, the distorted signal is coupled on to the main signal A. An envelope (video) modulator placed in the main signal path could give still more flexibility

Because of the nature of the klystron non-linearity, it is quite likely that all three non-linear effects described in Section 4 will not be eliminated simultaneously with one setting of the corrector. Further correction could be applied as envelope modulation. A suitable modulator could be driven by a detected video signal which had been modified by a gamma-corrector type of circuit. Such a modulation could eliminate crushing of sound and chrominance carriers by the vision carrier whilst the non-linear network in the alternative signal path would provide intermodulation products, variable in amplitude and phase to cancel with those produced by the klystron.

The advantage of this approach lies in the large number of parameters that may be varied and, thus the wide range of non-linear conditions for which it could compensate.

9. Possible use of an adaptive system

Long term drift of klystron and pre-corrector characteristics may limit the amount of correction achievable by cancellation of intermodulation products. For a high degree of correction necessary to run the klystron at high powers, this instability might be overcome by an automatic system whereby pre-corrector parameters were changed to optimise the performance.

Several problems would arise with such a system. The performance must be judged by comparing input and output signals and a suitable criterion signal produced. By a series of changes of pre-corrector parameters, the optimum setting of the system must be found for any condition of the klystron likely to occur. Whether or not the adjustments to parameters would be best made randomly or according to a pre-determined law would depend upon the nature of the criterion signal and how it was produced.

Any such system would be necessarily complex and expensive and it has yet to be decided whether the cost and effort would be justified in order to increase the usable powers of the klystron above those made possible by a pre-set system. Work by others would suggest that significant improvements in performance may be achieved without resorting to an adaptive system.

10. Conclusions

Exact pre-correction of the intrinsic non-linearity of the klystron is not possible. Nevertheless, a system permitting substantial power increases and improved efficiency is feasible using partial pre-correction.

The technique to be adopted is to distort signals before they are amplified in the klystron by means of a polynomial-type non-linearity which contains mainly third order and fifth order terms, suitably scaled. It is valid to replace actual television signals by the standard three-tone signal for purposes of calculation and measurement and on the basis of this test signal the levels of in-band intermodulation products produced in a klystron amplifier can be calculated. These calculated levels also agree well with measured levels.[3] Good approximations to the principal in-band intermodulation terms can be generated with a third and fifth order polynomial and to apply pre-correction a suitable radio frequency network must be devised exhibiting the required 'inverse' properties.

It may be necessary to include some degree of phase adjustment of products from the pre-corrector to ensure good cancellation of the principal intermodulation products. In addition some video envelope modulation may be considered to improve overall linearity.

It is understood that such pre-correction will be limited to use at powers still well below the saturation power of the klystron but possible increases in power of about 4 dB for combined sound and vision amplifiers is believed to be realistic. A system to produce this degree of improvement in performance would be highly desirable.

11. References

1. WEBSTER, D.L. 1939. Cathode ray bunching. *J. appl. Phys.*, 1939, 10, July, pp. 501 – 508.

2. WEBBER, S.E. 1958. Ballistic analysis of a two cavity finite beam klystron. *I.R.E. Trans. electronic Devices*, 1958, April, pp. 98 – 108.

3. EATON, J.L. *and* WHYTHE, D.J. 1967. The non-linear characteristics of klystron amplifiers. BBC Engng. Monogr., 1968, No. 74.

4. CHALK, C.O. *and* O'LOUGHLIN, C.N. 1968. Klystron amplifiers for television. *Electronic Equipment News*, 1968, Feb.

5. EDGCOMBE, C.J. *and* HEPPINSTALL, R. 1968. Non-linear behaviour in klystrons for UHF television. Nachrichtantech. Fachber, 1968, 35, pp. 151 – 156.

6. MURATA, S. 1964. Intermodulation produced by double cavity klystron. Tech. J. Japan Broadcasting Corporation, 1964, **16**, 5, pp. 1 – 8 (in Japanese).

7. O'LOUGHLIN, C.N. 1965. Intermodulation characteristics of high-power klystrons used in frequency transposers for colour television. International Conference on U.H.F. Television, London, Nov. 1965. I.E.R.E. Conference Proceedings, 1965, No. 6, Paper No. 10.

8. SAVAGE, D.C. *and* CARTER, D.A. 1964. Subjective impairment resulting from common amplification of modulated sound and colour television signals to U.K. 625-line standards. BBC Designs Department Technical Memorandum, 6.49 (64).

9. SIMS, G.D. *and* STEPHENSON, I.M. 1963. Microwave tubes and semi-conductor devices. London, Blackie, 1963.

10. MULLARD LTD. 1964. High-power klystrons for Band IV/V television transmitters. Technical Information 1, April 6, 1964.

APPENDIX A

Transfer Function of a Modulated Electron Beam

The velocity of an electron leaving the electron gun will be

$$u_o = (2\eta V_o)^{1/2} \qquad \text{A(1)}$$

Where η is the electronic charge/mass ratio ($1{\cdot}76 \times 10^{11}$ coulomb/kg).* A signal $V_1 x(t)$ applied across the input cavity gap ($-1 < x(t) < 1$) gives an electron passing the gap at time t a velocity $u(t)$ where

$$u(t) = u_o(1 + \Delta x(t)) \qquad \text{A(2)}$$

where $\Delta = \xi V_1 / V_o$

and ξ is the gap coupling factor, dependent upon the gap and beam geometry. Δ has a value considerably less than unity.

This assumes that the drift time of the electron across the gap is short enough for the changes in velocity to be considered to be impulsive. Such an assumption is reasonable as the transit time is less than 1 percent of a cycle for a signal at 500 MHz.

If the length of the drift tube is Z the time taken τ_1 for an electron starting at time t to arrive at the output gap will be

$$\tau_1 = Z/u(t) \qquad \text{A(3)}$$

or

$$\tau_1 = Z/u_o\{1 + \Delta x(t)\} \qquad \text{A(4)}$$

As $-1 < x(t) < 1$ term in Δ^2 and higher orders may be dropped

$$\therefore \qquad \tau_1 \simeq \frac{Z}{u_o}\left(1 - \Delta x(t)\right) \qquad \text{A(5)}$$

For an electron leaving the input gap at time δt later, the transit time will be

$$\tau_2 \simeq \frac{Z}{u_o}\left(1 - \Delta x(t + \delta t)\right) \qquad \text{A(6)}$$

Assuming to begin with, that by the end of the drift space, no fast electrons have overtaken slower ones and that the mutual repulsion between electrons may be neglected the incremental change on the electron beam δQ which passed the first cavity in time δt will pass the second cavity between times $(t + \tau_1)$ and $(t + \tau_2 + \delta t)$. (See Fig. 9.)

Let

$$\tau_2 + \delta t - \tau_1 = \delta\tau \qquad \text{A(7)}$$

Then from A(5) and A(6)

$$\frac{\delta Q}{\delta\tau} = \delta Q \Bigg/ \left(\delta t + \frac{Z\Delta}{u_o}\left\{x(t) - x(t + \delta t)\right\}\right) \qquad \text{A(8)}$$

Now

$$\underset{\delta t \to 0}{\text{Limit}} \; \frac{\delta Q}{\delta\tau} = \frac{dQ}{d\tau} = \frac{dQ/dt}{1 - \dfrac{Z}{u_o}\, x(t)} \qquad \text{A(9)}$$

Let

$$\tau + \frac{Z}{u_o} = t + \tau_1 \qquad \text{A(10)}$$

So from A(5)

$$\tau = t - \frac{Z\Delta}{u_o}\, x(t) \qquad \text{A(11)}$$

Now $dQ/d\tau$ in A(9) represents the current passing the output cavity at time τ and dQ/dt represents the current (d.c. in this case) passing the input cavity at time t. Neglecting the term Z/u_o in A(10) which merely represents a constant average delay down the drift tube we can write

$$I(\tau) = I_o / 1 - \frac{Z\Delta}{u_o}\, x(t) \qquad \text{A(12)}$$

Equation A(12) represents the transfer function of the electron beam in implicit form. A(11) shows the relationship between τ and t so the behaviour of the beam is completely described by A(11) and A(12) together. The frequency response of the input and output cavities will cause the overall behaviour of a klystron to vary considerably from this.

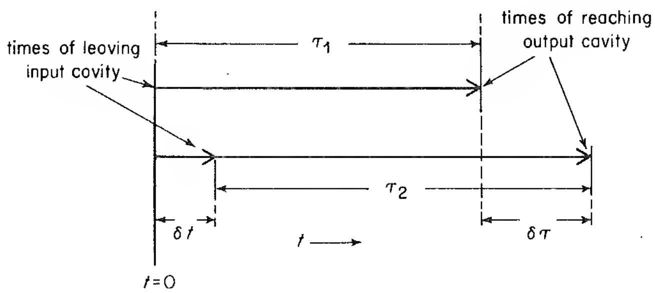

Fig. 9 - Electron transit times

* For a 10 kEV beam, relativisic effects cause an error of about 1·5 percent in Equation A(1).

APPENDIX B

Pre-correction Using an Inverse Characteristic

The non-linear effects of the klystron could be eliminated in principle by pre-distorting the input by means of a circuit having the inverse characteristic. Such a pre-correction could be achieved in two stages, corresponding to the two bunching effects shown in Figure 2 for a sinusoid. Firstly a circuit with the inverse characteristic of the amplitude function in Figure 2(*b*) could be employed, and secondly a phase distortion to counteract the effect shown in Figure 2(*a*). The first would involve a transfer function of the form

$$V_{out} = P\int\left\{1 - \frac{1}{Q(t)}\right\}dt \qquad B(1)$$

which can be seen to be the inverse of the expression contained in the modulus sign of Equation 4.

The required phase distortion would involve a delay line through which the propagation time depended upon the amplitude of the signal — thus

$$V_{out}(t) = V_{in}(t + RV_{in}(t)). \qquad B(2)$$

This approach would be fundamentally limited to amplitudes for which the time Equation A(11) has single valued solutions, corresponding to running the amplifier at levels below that at which over bunching occurs.

APPENDIX C

Synthesis of a Sinusoidal Wave at the Output of a Klystron

In order to produce a given output waveform at a high level from a klystron it is necessary to input a signal with a waveform such that the non-linearity of the klystron will distort the input signal into that which is required at the output.

Assuming, for simplicity, that a sinusoidal output is required then the current in the beam at the output will be

$$I(\tau) = I_0(1 + G\sin wt') \qquad C(1)$$

From Equation 4 we have

$$1 + G\sin(w\tau) = \sum_{tn}\left|\frac{1}{1 - K'\dot{V}_1(t_n)}\right| \qquad C(2)$$

where $V_1(t)$ is the input waveform

and $t_n - K'V_1(t_n) = \tau$ determines the values of t_n over which the RHS of C(2) is to be evaluated.

Provided $V_1(t)$ is restricted in amplitude so that there is only one value of t_n for each τ

$$1 + G\sin\left[\omega t - K'\omega V_1 t\right] = \frac{1}{1 - K'\dot{V}_1(t)} \qquad C(3)$$

$$\therefore \quad G\left[1 - K'V_1(t)\right]\sin\left[\omega(t - K'V_1(t)\right] = K'\dot{V}_1(t) \qquad C(4)$$

The LHS of Equation C(4) may be expressed as

$$-\frac{G}{\omega}\frac{d}{dt}\left\{\cos\left[\omega(t - K'V_1(t))\right]\right\} \qquad C(5)$$

so that

$$\cos\left[\omega(t - K'V_1(t)\right] = -\frac{K'\omega}{G}V_1(t) \qquad C(6)$$

The most convenient form of solution of Equation C(6) is the pair of parametric equations

$$V_1 = -\frac{G}{K'\omega}\cos\phi \qquad C(7)$$

and

$$\omega t = -G\cos\phi \qquad C(7A)$$

Figure 6 shows $V_1(wt)$ plotted for various values of G. For the equations $t - K'V_1(t) = t$ to have single-valued solutions G must not be greater than unity. This condition is also necessary to satisfy the requirement for non-reversal of beam current.

Of course, the high rates of change of input voltage shown by Fig. 5 to be necessary to produce a large output signal will be unattainable due to the frequency selective behaviour of the early cavities in the klystron.

SMW/JUC

Printed by BBC Research Department, Kingswood Warren, Tadworth, Surrey